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Superstrings in graviphoton background and $\mathcal{N} = \frac{1}{2} + \frac{3}{2}$ supersymmetry

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Abstract: Motivated by Ooguri and Vafa, we study superstrings in flat $\mathbb{R}^4$ in a constant self-dual graviphoton background. The supergravity equations of motion are satisfied in this background which deforms the $\mathcal{N} = 2$ $d = 4$ flat space super-Poincaré algebra to another algebra with eight supercharges. A D-brane in this space preserves a quarter of the supercharges; i.e. $\mathcal{N} = 1/2$ supersymmetry is realized linearly, and the remaining $\mathcal{N} = 3/2$ supersymmetry is realized nonlinearly. The theory on the brane can be described as a theory in noncommutative superspace in which the chiral fermionic coordinates θ^α of $\mathcal{N} = 1$ $d = 4$ superspace are not Grassman variables but satisfy a Clifford algebra.

Keywords: Superstrings and Heterotic Strings, D-branes, Superspaces.

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Contents

1. Introduction

Motivated by the work of Ooguri and Vafa [1] we continue the analysis of [2] of superstrings in flat $\mathbb{R}^4$ with $\mathcal{N}=2$ supersymmetry with constant self-dual graviphoton field strength.

Only in euclidean space can we turn on the self-dual part of the two form field strength $F^{\alpha\beta}$ while setting the anti-self-dual part $F^{\dot\alpha\dot\beta}$ to zero. Therefore we limit the discussion to euclidean space, but we will use lorentzian signature notation.[1]

The special property of a purely self-dual field strength $F^{\alpha\beta}$ is that it does not contribute to the energy momentum tensor, and therefore does not lead to a source in the gravity equations of motion. Also, since the kinetic term of the graviphoton does not depend on the dilaton, the background $F^{\alpha\beta}$ does not lead to a source in the dilaton equation. Therefore, this background is a solution of the equations of motion.

Before turning on the graviphoton our system has $\mathcal{N}=2$ supersymmetry. What happens to it as a result of the nonzero $F^{\alpha\beta}$? The local $\mathcal{N}=2$ $d=4$ chiral gravitini $\xi_\mu^{\beta j}$ transformation laws are

$$\delta\xi_\mu^{\beta j}=\sigma_{\mu\alpha\dot\alpha}\overline{\varepsilon}^{\dot\alpha j}\alpha' F^{\alpha\beta}+\nabla_\mu\varepsilon^{\beta j} \tag{1.1}$$

where $\varepsilon^{\beta j}$ and $\varepsilon^{\dot\beta j}$ are the local supersymmetry parameters. The dilatini and antichiral gravitini do not transform into the self-dual graviphoton field strength. The variation (1.1) clearly vanishes for constant $\varepsilon^{\beta j}$ and $\overline{\varepsilon}^{\dot\alpha j}=0$. This shows that the background $F^{\alpha\beta}$ does not affect the four supercharges with one chirality $Q_{\alpha i}$. The variation (1.1) is not zero for constant $\overline{\varepsilon}^{\dot\alpha j}$ showing that the four other standard supercharges are broken by $F^{\alpha\beta}$. However, one can easily make the variation of the chiral gravitini (1.1) vanish if one chooses

$$\varepsilon^{\beta j}=\frac{1}{2}\alpha' F^{\alpha\beta}\sigma_{\mu\alpha\dot\alpha}\overline{\varepsilon}^{\dot\alpha j}x^\mu \tag{1.2}$$

where $\overline{\varepsilon}^{\dot\alpha j}$ is constant. So in the presence of this background, one has eight global fermionic symmetries which generate a deformed $\mathcal{N}=2$ $d=4$ supersymmetry.

[1] We will use the conventions of Wess and Bagger [3].

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One can easily read off the algebra of the deformed $\mathcal{N} = 2$ $d = 4$ supersymmetry from the above transformations

$$\begin{aligned}
\{Q_j^\alpha, \overline{Q}_k^{\dot\alpha}\} &= 2\epsilon_{jk}\sigma_\mu^{\alpha\dot\alpha}P^\mu \\
[P_\mu, Q_{\alpha j}] &= [P_\mu, P_\nu] = \{Q_{\alpha j}, Q_{\beta k}\} = 0 \\
[P_\mu, \overline{Q}_{\dot\alpha j}] &= 2\sigma_{\mu\beta\dot\alpha}\alpha' F^{\alpha\beta}Q_{\alpha j} \\
\{\overline{Q}_{\dot\alpha j}, \overline{Q}_{\dot\beta k}\} &= 4\epsilon_{jk}\epsilon_{\dot\alpha\dot\beta}\alpha' F^{\alpha\beta}M_{\alpha\beta}
\end{aligned} \tag{1.3}$$

where $M_{\alpha\beta}$ is the self-dual Lorentz generator which is normalized to transform

$$[M_{\alpha\beta}, \theta^\gamma] = \theta_\alpha\delta_\beta^\gamma + \theta_\beta\delta_\alpha^\gamma\,. \tag{1.4}$$

Note that if one had turned on both a self-dual field strength $F^{\alpha\beta}$ and an anti-self-dual field strength $F^{\dot\alpha\dot\beta}$, there would have been a backreaction which would warp the spacetime to (euclidean) $AdS_2 \times S^2$. The isometries of this space form a $PSU(2|2)$ algebra, and the algebra of (1.3) can be understood as a certain contraction of $PSU(2|2)$. In [4], the superstring was studied in this $AdS_2 \times S^2$ background and the structure of the worldsheet action closely resembles the pure spinor version of the $AdS_5 \times S^5$ worldsheet action. Since the worldsheet action becomes quadratic in the limit where the anti-self-dual field strength $F^{\dot\alpha\dot\beta}$ goes to zero, this limit might be useful for studying $AdS_d \times S^d$ backgrounds in a manner analogous to the Penrose limit.

In the next section we will extend this supergravity discussion to superstring theory. Using the hybrid formalism for the superstring, the worldsheet action remains quadratic in this graviphoton background, so the theory is trivial to analyze. We will explicitly show that it preserves eight supercharges with the new algebra (1.3). In section 3 we will study D-branes in this background, and will examine the symmetries on the branes. In section 4 we will consider the low energy field theory on the D-brane and will examine its supersymmetries.

2. Superstrings in graviphoton background

In this section and the next one we review and extend the discussion in [1, 2] of type-II superstrings in $\mathbb{R}^4$ with $\mathcal{N} = 2$ supersymmetry deformed by a self-dual graviphoton (see also [5]).

Since we are interested in superstrings in Ramond-Ramond background, the standard NSR formalism cannot be used. Instead, we will use the hybrid formalism (for a review see [6]). The relevant part of the worldsheet lagrangian is

$$\mathcal{L} = \frac{1}{\alpha'}\left(\frac{1}{2}\widetilde{\partial}x^\mu\partial x_\mu + p_\alpha\widetilde{\partial}\theta^\alpha + \overline{p}_{\dot\alpha}\widetilde{\partial}\overline{\theta}^{\dot\alpha} + \widetilde{p}_\alpha\partial\widetilde{\theta}^\alpha + \widetilde{\overline{p}}_{\dot\alpha}\partial\widetilde{\overline{\theta}}^{\dot\alpha}\right), \tag{2.1}$$

where $\mu = 0, \ldots, 3$, $(\alpha, \dot\alpha) = 1, 2$, and we ignore the worldsheet fields ρ and the Calabi-Yau sector. Since we use a bar to denote the space time chirality, we use a tilde to denote the worldsheet chirality. Therefore when the worldsheet has euclidean signature it is parametrized by z and $\widetilde{z}$ which are complex conjugate of each other. p, $\widetilde{p}$, $\overline{p}$ and $\widetilde{\overline{p}}$ are canonically conjugate to θ, $\widetilde{\theta}$ $\overline{\theta}$ and $\widetilde{\overline{\theta}}$; they are the worldsheet versions of $-\frac{\partial}{\partial\theta}|_x$ etc.. The reason x is held fixed in these derivatives is that x appears as another independent field in (2.1).

It is convenient to change variables to

$$
\begin{aligned}
y^\mu &= x^\mu + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha} + i\widetilde{\theta}^\alpha\sigma^\mu_{\alpha\dot\alpha}\widetilde{\overline{\theta}}^{\dot\alpha} \\
\overline{d}_{\dot\alpha} &= \overline{p}_{\dot\alpha} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial x_\mu - \theta\theta\partial\overline{\theta}_{\dot\alpha} + \frac{1}{2}\overline{\theta}_{\dot\alpha}\partial(\theta\theta) \\
q_\alpha &= -p_\alpha - i\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\partial x_\mu + \frac{1}{2}\overline{\theta\theta}\partial\theta_\alpha - \frac{3}{2}\partial(\theta_\alpha\overline{\theta\theta}) \\
\widetilde{\overline{d}}_{\dot\alpha} &= \widetilde{\overline{p}}_{\dot\alpha} - i\widetilde{\theta}^\alpha\sigma^\mu_{\alpha\dot\alpha}\widetilde{\partial} x_\mu - \widetilde{\theta}\widetilde{\theta}\widetilde{\partial}\widetilde{\overline{\theta}}_{\dot\alpha} + \frac{1}{2}\widetilde{\overline{\theta}}_{\dot\alpha}\widetilde{\partial}(\widetilde{\theta}\widetilde{\theta}) \\
\widetilde{q}_\alpha &= -\widetilde{p}_\alpha - i\sigma^\mu_{\alpha\dot\alpha}\widetilde{\overline{\theta}}^{\dot\alpha}\widetilde{\partial} x_\mu + \frac{1}{2}\widetilde{\overline{\theta}}\widetilde{\overline{\theta}}\widetilde{\partial}\widetilde{\theta}_\alpha - \frac{3}{2}\widetilde{\partial}(\widetilde{\theta}_\alpha\widetilde{\overline{\theta}}\widetilde{\overline{\theta}})
\end{aligned}
\tag{2.2}
$$

and to derive

$$
\mathcal{L} = \frac{1}{\alpha'}\left(\frac{1}{2}\widetilde{\partial} y^\mu\partial y_\mu - q_\alpha\widetilde{\partial}\theta^\alpha + \overline{d}_{\dot\alpha}\widetilde{\partial}\overline{\theta}^{\dot\alpha} - \widetilde{q}_\alpha\partial\widetilde{\theta}^\alpha + \widetilde{\overline{d}}_{\dot\alpha}\partial\widetilde{\overline{\theta}}^{\dot\alpha} + \text{total derivative}\right). \tag{2.3}
$$

The new variables in (2.2) are the worldsheet versions of $\overline{D}_{\dot\alpha} = -\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}}|_y$, $Q_\alpha = \frac{\partial}{\partial\theta^\alpha}|_y$, etc.. The reason y is held fixed in these derivatives is that it appears as an independent field in (2.3). Our definitions of q and $\widetilde{q}$ differ from the integrand of the supercharges in [6] by total derivatives which do not affect the charges, but are important for our purpose. We will also need the worldsheet versions of D_α, $\widetilde{D}_\alpha$ and the other two supercharges

$$
\begin{aligned}
d_\alpha &= -p_\alpha + i\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\partial x_\mu - \overline{\theta\theta}\partial\theta_\alpha + \frac{1}{2}\theta_\alpha\partial(\overline{\theta\theta}) = q_\alpha + 2i\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\partial y_\mu - 4\overline{\theta\theta}\partial\theta_\alpha \\
\widetilde{d}_\alpha &= -\widetilde{p}_\alpha + i\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\widetilde{\partial} x_\mu - \widetilde{\overline{\theta}}\widetilde{\overline{\theta}}\widetilde{\partial}\widetilde{\theta}_\alpha + \frac{1}{2}\widetilde{\theta}_\alpha\widetilde{\partial}(\widetilde{\overline{\theta}}\widetilde{\overline{\theta}}) = \widetilde{q}_\alpha + 2i\sigma^\mu_{\alpha\dot\alpha}\widetilde{\overline{\theta}}^{\dot\alpha}\widetilde{\partial} y_\mu - 4\widetilde{\overline{\theta}}\widetilde{\overline{\theta}}\widetilde{\partial}\widetilde{\theta}_\alpha \\
\overline{q}_{\dot\alpha} &= \overline{p}_{\dot\alpha} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial x_\mu + \frac{1}{2}\theta\theta\partial\overline{\theta}_{\dot\alpha} - \frac{3}{2}\partial(\overline{\theta}_{\dot\alpha}\theta\theta) \\
\widetilde{\overline{q}}_{\dot\alpha} &= \widetilde{\overline{p}}_{\dot\alpha} + i\widetilde{\theta}^\alpha\sigma^\mu_{\alpha\dot\alpha}\widetilde{\partial} x_\mu + \frac{1}{2}\widetilde{\theta}\widetilde{\theta}\widetilde{\partial}\widetilde{\overline{\theta}}_{\dot\alpha} - \frac{3}{2}\widetilde{\partial}(\widetilde{\overline{\theta}}_{\dot\alpha}\widetilde{\theta}\widetilde{\theta}).
\end{aligned}
\tag{2.4}
$$

Since the vertex operator for a constant self-dual graviphoton field strength is

$$
\int d^2z\, q_\alpha\widetilde{q}_\beta F^{\alpha\beta} \tag{2.5}
$$

in the hybrid formalism, the action remains quadratic in this background and there is no backreaction. The lagrangian is

$$
\mathcal{L} = \frac{1}{\alpha'}\left(\frac{1}{2}\widetilde{\partial} y^\mu\partial y_\mu - q_\alpha\widetilde{\partial}\theta^\alpha + \overline{d}_{\dot\alpha}\widetilde{\partial}\overline{\theta}^{\dot\alpha} - \widetilde{q}_\alpha\partial\widetilde{\theta}^\alpha + \widetilde{\overline{d}}_{\dot\alpha}\partial\widetilde{\overline{\theta}}^{\dot\alpha} + \alpha' F^{\alpha\beta}q_\alpha\widetilde{q}_\beta\right). \tag{2.6}
$$

The fields q and $\widetilde{q}$ can easily be integrated out using their equations of motion

$$
\widetilde{\partial}\theta^\alpha = \alpha' F^{\alpha\beta}\widetilde{q}_\beta \;\; \partial\widetilde{\theta}^\alpha = -\alpha' F^{\alpha\beta}q_\beta\,. \tag{2.7}
$$

Let us examine the supersymmetries of the lagrangian (2.6). In addition to being invariant under the Q_α and $\widetilde{Q}_\alpha$ supersymmetries generated by

$$\delta\theta^\alpha = \varepsilon^\alpha\,, \qquad \delta\widetilde{\theta}^\alpha = \widetilde{\varepsilon}^\alpha\,, \tag{2.8}$$

where ε^α and $\widetilde{\varepsilon}^\alpha$ are constants, the above lagrangian is also invariant (up to total derivatives) under the $\overline{Q}_{\dot\alpha}$ and $\overline{\widetilde{Q}}_{\dot\alpha}$ supersymmetries generated by

$$\begin{aligned}
\delta\overline{\theta}^{\dot\alpha} &= -\overline{\varepsilon}^{\dot\alpha} \\
\delta\overline{\widetilde{\theta}}^{\dot\alpha} &= -\overline{\widetilde{\varepsilon}}^{\dot\alpha} \\
\delta y^\mu &= 2i\sigma^\mu_{\alpha\dot\alpha}(\overline{\varepsilon}^{\dot\alpha}\theta^\alpha + \overline{\widetilde{\varepsilon}}^{\dot\alpha}\widetilde{\theta}^\alpha) \\
\delta q_\alpha &= 2i\overline{\varepsilon}^{\dot\alpha}\sigma_{\mu\alpha\dot\alpha}\partial y^\mu \\
\delta\widetilde{q}_\alpha &= 2i\overline{\widetilde{\varepsilon}}^{\dot\alpha}\sigma_{\mu\alpha\dot\alpha}\widetilde{\partial} y^\mu \\
\delta\theta^\alpha &= 2i\alpha' F^{\alpha\beta}\overline{\widetilde{\varepsilon}}^{\dot\alpha} y^\mu\sigma_{\mu\beta\dot\alpha} \\
\delta\widetilde{\theta}^\alpha &= -2i\alpha' F^{\alpha\beta}\overline{\varepsilon}^{\dot\alpha} y^\mu\sigma_{\mu\beta\dot\alpha}\,.
\end{aligned} \tag{2.9}$$

These transformations are the stringy version of the supergravity variation (1.2). The last two transformations in (2.9) represent the deformation, and are needed because of the deformation term $F^{\alpha\beta}q_\alpha\widetilde{q}_\beta$ in the lagrangian. Unlike the other transformations in (2.9), the last two depend explicitly on y^μ. This fact is similar to the explicit x dependence in (1.2). Therefore, the corresponding currents $\overline{q}_{\dot\alpha}$ and $\overline{\widetilde{q}}_{\dot\alpha}$ are not holomorphic and anti-holomorphic respectively. They are similar to the currents of the Lorentz symmetries in the worldsheet theory.

In order to establish that (2.9) are not only symmetries of the worldsheet lagrangian but are also symmetries of the string, we should check that they commute with the BRST operators. To do that, we use d_α and $\widetilde{d}_\alpha$ of (2.4) to show that under (2.9)

$$\delta d_\alpha = 8\left(\overline{\widetilde{\varepsilon}}_{\dot\alpha}\overline{\theta}^{\dot\alpha}\right)\epsilon_{\alpha\gamma}\alpha' F^{\gamma\beta}d_\beta\,, \qquad \delta\widetilde{d}_\alpha = -8\left(\overline{\varepsilon}_{\dot\alpha}\overline{\widetilde{\theta}}^{\dot\alpha}\right)\epsilon_{\alpha\gamma}\alpha' F^{\gamma\beta}\widetilde{d}_\beta, \tag{2.10}$$

where we have used the equations of motion (2.7). Using (2.10) it is clear that the BRST operators

$$\begin{aligned}
G &= d_\alpha d^\alpha e^\rho\,, \qquad & \overline{G} &= \overline{d}_{\dot\alpha}\overline{d}^{\dot\alpha}e^{-\rho} \\
\widetilde{G} &= \widetilde{d}_\alpha\widetilde{d}^\alpha e^{\widetilde{\rho}}\,, \qquad & \overline{\widetilde{G}} &= \overline{\widetilde{d}}_{\dot\alpha}\overline{\widetilde{d}}^{\dot\alpha}e^{-\widetilde{\rho}}\,,
\end{aligned} \tag{2.11}$$

are invariant. This establishes that (2.9) are symmetries of the theory.

One can easily read off the algebra of the deformed $\mathcal{N}=2$ $d=4$ supersymmetry from the above transformations. Identifying $(Q_\alpha, \widetilde{Q}_\alpha)$ with $(Q_{\alpha 1}, Q_{\alpha 2})$ and $(\overline{Q}_{\dot\alpha}, \overline{\widetilde{Q}}_{\dot\alpha})$ with $(\overline{Q}_{\dot\alpha 2}, -\overline{\widetilde{Q}}_{\dot\alpha 1})$, one obtains the algebra (1.3).

3. *D*-branes

Consider first the system without the background graviphoton. If the worldsheet ends on a D-brane, the boundary conditions are easily found by imposing that there is no surface term in the equations of motion. For a boundary at $z=\widetilde{z}$, we can use the boundary conditions

$\theta(z = \widetilde{z}) = \widetilde{\theta}(z = \widetilde{z})$, $q(z = \widetilde{z}) = \widetilde{q}(z = \widetilde{z})$, etc. Then the solutions of the equations of motion are such that $\theta(z) = \widetilde{\theta}(\widetilde{z})$, $q(z) = \widetilde{q}(\widetilde{z})$, etc.; i.e. the fields extend to holomorphic fields beyond the boundary. The boundary breaks half the supersymmetries preserving only $\oint q dz + \oint \widetilde{q} d\widetilde{z}$ and $\oint \overline{q} dz + \oint \widetilde{\overline{q}} d\widetilde{z}$. From a spacetime point of view, these unbroken supersymmetries are realized linearly on the brane, while the other supersymmetries, which are broken by the boundary conditions, are realized nonlinearly.

Now, let us consider the system with nonzero $F^{\alpha\beta}$. After integrating out q and $\widetilde{q}$ the relevant part of the worldsheet lagrangian is

$$\mathcal{L}_{\text{eff}} = \left(\frac{1}{\alpha'^2 F}\right)_{\alpha\beta} \partial\widetilde{\theta}^\alpha \widetilde{\partial}\theta^\beta \,. \tag{3.1}$$

The appropriate boundary conditions are

$$\theta^a(z = \widetilde{z}) = \widetilde{\theta}^a(z = \widetilde{z})\,, \qquad \partial\widetilde{\theta}^a(z = \widetilde{z}) = -\widetilde{\partial}\theta^a(z = \widetilde{z})\,. \tag{3.2}$$

The first condition states that the superspace has half the number of θs. The second condition guarantees, using (2.7) that $q_\alpha(z = \widetilde{z}) = \widetilde{q}_\alpha(z = \widetilde{z})$.

It is clear that the supercharges $\oint q dz + \oint \widetilde{q} d\widetilde{z}$ are preserved by the boundary conditions and are still realized linearly on the brane. However, the supercharges $\oint \overline{q} dz + \oint \widetilde{\overline{q}} d\widetilde{z}$ are broken and are realized nonlinearly.

The propagator of θ is found to be

$$\langle \theta^\alpha(z, \widetilde{z}) \theta^\beta(w, \widetilde{w}) \rangle = \frac{\alpha'^2 F^{\alpha\beta}}{2\pi i} \log \frac{\widetilde{z} - w}{z - \widetilde{w}} \tag{3.3}$$

with the branch cut of the logarithm outside the worldsheet. Therefore for two points on the boundary $z = \widetilde{z} = \tau$ and $w = \widetilde{w} = \tau'$

$$\langle \theta^\alpha(\tau) \theta^\beta(\tau') \rangle = \frac{\alpha'^2 F^{\alpha\beta}}{2} \text{sign}(\tau - \tau')\,. \tag{3.4}$$

Using standard arguments about open string coupling, this leads to

$$\{\theta^\alpha, \theta^\beta\} = \alpha'^2 F^{\alpha\beta} = C^{\alpha\beta} \neq 0\,; \tag{3.5}$$

i.e. to a deformation of the anticommutator of the θs. It is important that since the coordinates $\overline{\theta}$ and y were not affected by the background coupling (2.5), they remain commuting. In particular we derive that $[y^\mu, y^\nu] = [y^\mu, \theta^\alpha] = 0$, and therefore $[x^\mu, x^\nu] \neq 0$ and $[x^\mu, \theta^\alpha] \neq 0$ [2].

For a partial list of other references on noncommuting θs see [7]–[19].

4. The zero slope limit

The theory on the branes is simplified in the zero slope limit $\alpha' \to 0$. If we want to preserve the nontrivial anticommutator of the θs (3.5), we should scale

$$\alpha' \to 0\,, \qquad F^{\alpha\beta} \to \infty\,, \qquad \alpha'^2 F^{\alpha\beta} = C^{\alpha\beta} = \text{fixed}\,. \tag{4.1}$$

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For $C = 0$ the low energy field theory on the branes is super-Yang-Mills theory. For nonzero C it is [2]

$$S = S_0 - i \int d^4x C^{\mu\nu}\,\mathrm{Tr}(v_{\mu\nu}\overline{\lambda\lambda}) + \frac{1}{4}\int d^4x C^{\mu\nu}C_{\mu\nu}\,\mathrm{Tr}(\overline{\lambda\lambda})^2 \tag{4.2}$$

where $S_0 = \int d^4x Tr[-\frac{1}{2}v^{\mu\nu}v_{\mu\nu} - 2i\lambda\sigma^m\overline{\lambda} + D^2]$ is the usual super-Yang-Mills action, $v^{\mu\nu}$ is the Yang-Mills field strength, $\overline{\lambda}^{\dot{\alpha}}$ is the antichiral gaugino, and $C^{\mu\nu} = (\sigma^{\mu\nu})_{\alpha\beta}C^{\alpha\beta}$.

We will now examine the supersymmetries of the action (4.2).

Under the $\mathcal{N} = 1/2$ supersymmetries generated by $Q_\alpha + \widetilde{Q}_\alpha$, the chiral superfield W^α transforms as

$$\delta W^\beta = \eta_+^\alpha \frac{\partial}{\partial\theta^\alpha}W^\beta \tag{4.3}$$

where η_+^α is the constant supersymmetry parameter. In component fields, these transformations are [2]

$$\begin{aligned}
\delta\lambda &= i\eta_+ D + \sigma^{\mu\nu}\eta_+\left(v_{\mu\nu} + \frac{i}{2}C_{\mu\nu}\overline{\lambda\lambda}\right)\\
\delta v_{\mu\nu} &= i\eta_+(\sigma_\nu\nabla_\mu - \sigma_\mu\nabla_\nu)\overline{\lambda}\\
\delta D &= -\eta_+\sigma^\mu\nabla_\mu\overline{\lambda}\\
\delta\overline{\lambda} &= 0\,.
\end{aligned} \tag{4.4}$$

The action of (4.2) is invariant under these transformations after including the $C_{\mu\nu}$-dependent terms in the transformations and in the action.

However, under the $\mathcal{N} = 3/2$ supersymmetries corresponding to $(Q_\alpha - \widetilde{Q}_\alpha, \overline{Q}_{\dot{\alpha}}, \widetilde{\overline{Q}}_{\dot{\alpha}})$, the component fields transform nonlinearly since these transformations do not preserve the D-brane boundary conditions $\theta^\alpha = \widetilde{\theta}^\alpha$ and $\overline{\theta}^{\dot{\alpha}} = \widetilde{\overline{\theta}}^{\dot{\alpha}}$.[2] To determine the nonlinear transformation of these component fields, it is useful to recall that in a background with abelian chiral and antichiral superfields W^α and $\overline{W}^{\dot{\alpha}}$, the D-brane boundary conditions $\theta^\alpha - \widetilde{\theta}^\alpha = 0$ and $\overline{\theta}^{\dot{\alpha}} - \widetilde{\overline{\theta}}^{\dot{\alpha}} = 0$ are modified to $\theta^\alpha - \widetilde{\theta}^\alpha = \frac{1}{4}\alpha' W^\alpha$ and $\overline{\theta}^{\dot{\alpha}} - \widetilde{\overline{\theta}}^{\dot{\alpha}} = \frac{1}{4}\alpha'\overline{W}^{\dot{\alpha}}$ to leading order in α' [20, 21, 22]. Just as the bosonic D-brane boundary conditions $\partial_\sigma x_\mu = 0$ is modified to $\partial_\sigma x_\mu = \alpha'(\partial_\mu A_\nu - \partial_\nu A_\mu)\partial_\tau x^\nu$ in the presence of the Maxwell vertex operator $\int d\tau A_\mu(x)\partial_\tau x^\mu$ [23, 24], the superstring D-brane boundary conditions are modified to $\theta^\alpha - \widetilde{\theta}^\alpha = \frac{1}{4}\alpha' W^\alpha$ and $\overline{\theta}^{\dot{\alpha}} - \widetilde{\overline{\theta}}^{\dot{\alpha}} = \frac{1}{4}\alpha'\overline{W}^{\dot{\alpha}}$ in the presence of the super-Maxwell vertex operator which includes the term $\int d\tau(W^\alpha d_\alpha + \overline{W}^{\dot{\alpha}}\overline{d}_{\dot{\alpha}})$ [20]. So by comparing with the supersymmetry transformations of $(\theta, \overline{\theta}, \widetilde{\theta}, \widetilde{\overline{\theta}})$ in (2.8)(2.9), one learns the nonlinear transformations of the abelian gauginos λ^α and $\overline{\lambda}^{\dot{\alpha}}$ to leading order in α'. Note that the nonlinear transformations of all other super-Yang-Mills fields are higher order in α'.

For example, under $Q_\alpha - \widetilde{Q}_\alpha$, $\delta(\theta - \widetilde{\theta}) = \varepsilon - \widetilde{\varepsilon}$ in (2.9) implies that the chiral abelian gaugino transforms inhomogeneously as

$$\delta\lambda^\alpha = \frac{4i}{\alpha'}(\varepsilon^\alpha - \widetilde{\varepsilon}^\alpha) \equiv \eta_-^\alpha\,. \tag{4.5}$$

[2] Actually, since these supersymmetries are *spontaneously* broken on the D-brane, the corresponding supercharges $(Q_\alpha - \widetilde{Q}_\alpha, \overline{Q}_{\dot{\alpha}}, \widetilde{\overline{Q}}_{\dot{\alpha}})$ do not exist. We will denote by $(Q_\alpha - \widetilde{Q}_\alpha, \overline{Q}_{\dot{\alpha}}, \widetilde{\overline{Q}}_{\dot{\alpha}})$ the transformations of the supersymmetries.

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This transformation clearly leaves invariant the action of (4.2). In the zero slope limit we should take $\varepsilon - \widetilde{\varepsilon} \to 0$ with fixed η_- in order to have a finite answer.

Under $\overline{Q}_{\dot\alpha} - \widetilde{\overline{Q}}_{\dot\alpha}$, the antichiral abelian gaugino transforms inhomogeneously as

$$\delta\overline{\lambda}^{\dot\alpha} = \frac{4i}{\alpha'}(\overline{\varepsilon}^{\dot\alpha} - \widetilde{\overline{\varepsilon}}^{\dot\alpha}) \equiv \overline{\eta}_-^{\dot\alpha}\,, \tag{4.6}$$

and therefore in the zero slope limit we should take $\overline{\varepsilon} - \widetilde{\overline{\varepsilon}} \to 0$ with fixed $\overline{\eta}_-$ in order to have a finite answer. In addition to (4.6), since $\delta(\theta^\alpha + \widetilde{\theta}^\alpha) = -\frac{1}{2}\overline{\eta}_-^{\dot\alpha} C^{\alpha\beta} y_\mu \sigma^\mu_{\beta\dot\alpha}$ is finite in the zero slope limit, the chiral superfield W^γ transforms as

$$\delta W^\gamma = -\frac{1}{2}\overline{\eta}_-^{\dot\alpha} C^{\alpha\beta} y_\mu \sigma^\mu_{\beta\dot\alpha} \frac{\partial}{\partial\theta^\alpha} W^\gamma\,. \tag{4.7}$$

Therefore, to leading order in α', the component fields transform as

$$\begin{aligned}
\delta\lambda &= i\eta(y,\overline{\eta}_-)D + \sigma^{\mu\nu}\eta(y,\overline{\eta}_-)(v_{\mu\nu} + \frac{i}{2}C_{\mu\nu}\overline{\lambda\lambda})\\
\delta v_{\mu\nu} &= i\eta(y,\overline{\eta}_-)(\sigma_\nu\nabla_\mu - \sigma_\mu\nabla_\nu)\overline{\lambda}\\
\delta D &= -\eta(y,\overline{\eta}_-)\sigma^\mu\nabla_\mu\overline{\lambda}\\
\delta\overline{\lambda} &= \overline{\eta}_-\,.
\end{aligned} \tag{4.8}$$

where $\eta(y,\overline{\eta}_-)^\alpha \equiv -\frac{1}{2}\overline{\eta}_-^{\dot\alpha} C^{\alpha\beta} y_\mu \sigma^\mu_{\beta\dot\alpha}$.

To explicitly check that (4.2) is invariant under the transformation of (4.8), first note that under the inhomogeneous transformation of $\overline{\lambda}$,

$$\delta_{\text{inhomogeneous}} S = -2i\int d^4x C^{\mu\nu}\,\mathrm{Tr}\left[v_{\mu\nu}\overline{\eta}_-\overline{\lambda}\right] + \int d^4x C^{\mu\nu}C_{\mu\nu}\,\mathrm{Tr}\left[(\overline{\eta}_-\overline{\lambda})(\overline{\lambda\lambda})\right]. \tag{4.9}$$

And under the y-dependent transformation parameterized by $\eta(y,\overline{\eta}_-)$ in (4.8),

$$\delta_{\text{homogeneous}} S = \int d^4x J^\alpha_\mu \frac{\partial}{\partial y_\mu}\eta(y,\overline{\eta}_-)_\alpha \tag{4.10}$$

where J^α_μ is the conserved supersymmetry current associated with the invariance of (4.2) under the global supersymmetry transformation of (4.4). One can easily compute that

$$J_{\mu\ \alpha} = \mathrm{Tr}\left[-2iv^{\rho\tau}(\sigma_{\rho\tau})_\alpha{}^\beta\sigma_{\mu\ \beta\dot\gamma}\overline{\lambda}^{\dot\gamma} + 2C_{\mu\nu}\sigma^\nu_{\alpha\dot\alpha}\overline{\lambda}^{\dot\alpha}(\overline{\lambda\lambda})\right] \tag{4.11}$$

and

$$\frac{\partial}{\partial y^\mu}\eta(y,\overline{\eta}_-)_\alpha = -\frac{1}{2}(\overline{\eta}_-)_{\dot\alpha}C_{\alpha\beta}\sigma^{\beta\dot\alpha}_\mu\,. \tag{4.12}$$

Using (4.11) and (4.12) in (4.10), one finds that (4.10) cancels (4.9) so the action is invariant.

Lastly, under $\overline{Q}_{\dot\alpha} + \widetilde{\overline{Q}}_{\dot\alpha}$, the chiral abelian gaugino transforms inhomogeneously as

$$\delta\lambda^\alpha = -8(\overline{\varepsilon}^{\dot\alpha} + \widetilde{\overline{\varepsilon}}^{\dot\alpha})(\alpha')^{-2}C^{\alpha\beta}y_\mu\sigma^\mu_{\beta\dot\alpha} \equiv -8\overline{\eta}_+^{\dot\alpha}C^{\alpha\beta}y_\mu\sigma^\mu_{\beta\dot\alpha}\,. \tag{4.13}$$

Note that in this case we need $\overline{\varepsilon}^{\dot\alpha} + \widetilde{\overline{\varepsilon}}^{\dot\alpha} \sim \alpha'^2$ in order to have a finite transformation in the zero slope limit. Since $\delta(\overline{\theta}^{\dot\alpha} + \widetilde{\overline{\theta}}^{\dot\alpha}) = (\overline{\varepsilon}^{\dot\alpha} + \widetilde{\overline{\varepsilon}}^{\dot\alpha}) \sim \alpha'^2$, we should set it to zero in the zero slope limit. Therefore, under this transformation (4.13) is the only variation. To check invariance of (4.2) under (4.13), note that the inhomogeneous transformation of λ^α leaves the action invariant since $\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\mu}\delta\lambda^\alpha = 0$.

We conclude that the action (4.2) is invariant under the linear $\mathcal{N} = \frac{1}{2}$ supersymmetry transformations (4.3)(4.4) with parameter η_+ and under the nonlinear $\mathcal{N} = \frac{3}{2}$ supersymmetry transformations (4.5)(4.8)(4.13) with parameters η_-, $\overline{\eta}_-$, $\overline{\eta}_+$.

It would be interesting to extend our analysis to higher-order terms in α'. This amounts to finding a noncommutative superspace generalization to the $d = 4$ supersymmetric Born-Infeld action. Just as the standard supersymmetric Born-Infeld action realizes $\mathcal{N} = 1 + 1$ supersymmetry (four supercharges are realized linearly and four supercharges are realized nonlinearly) [21, 22], the noncommutative superspace Born-Infeld theory should realize $\mathcal{N} = \frac{1}{2} + \frac{3}{2}$ supersymmetry.

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